Geant 4 simulation of the DEPFET beam test Internal Note

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Version 1

Daniel Scheirich, Peter Kodys,
Zdenek Dolezal, Pavel Reznicek

Institute of Particle and Nuclear Physics,
Faculty of Mathematics and Physiscs,
Charles University, Prague

This is an internal note that will be part of the Bachelor Thesis of Daniel Scheirich.

1 Introduction

In this study we present some preliminary results of a beam test simulation in the Geant 4. For the beam tests with a low energy electron beam a multiple scattering could be a serious problem. Hence we made simulations of several beam test geometries in order to find an optimal one. Some different contributions to the final beam test resolution are discussed in this paper.

2 Model validation

In order to have a crosscheck of simulation results a well known problem was simulated firstly. An electron scattering in the single silicon wafer was simulated to obtain an angular distribution.

According to Eidelman *et Al.* [1] a theoretical shape of the distribution is described by the Moliere distribution function. It's approximately Gaussian for small angles, but for angles greater than few ϑ_0 (defined below) it has larger tails than Gaussian does.

It is sufficient to use a Gaussian approximation for the central 98% of the projected angular distribution, with a width ϑ_0 defined in [1]:

$$\theta_0 = \frac{13.6\text{MeV}\, z}{\beta c p} \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \ln\left(\frac{x}{X_0}\right)\right), \tag{1}$$

where p, βc and z are the momentum, velocity and charge number, and x/X_0 is the thickness in the radiation length (defined in [1]).

A geometry of the simulation is shown in Fig. 1. A thickness of a silicon wafer is $300\,\mu$m and its center is placed in the center of the coordinate system. Primary electrons move along the x-axis. ϑ is an angle between original and a new direction of a scattered electron. The whole problem is three-dimensional, z-axis is not drawn in the figure.

The simulation was done for 1 GeV to 5 GeV electrons, 50000 events for each run. Positions of verticies of electron tracks were saved to the output file. Sample of 100 tracks in the x-y plane is shown in Fig. 2. Only the tracks of primary particles were saved (no δ-electrons). Each track consist of single lines (steps). The track in silicon and one more step in air for each particle was saved.

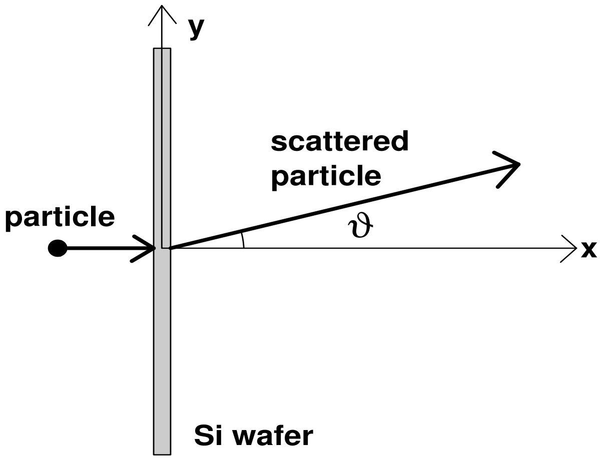


Figure 1: the geometry of the simulation.

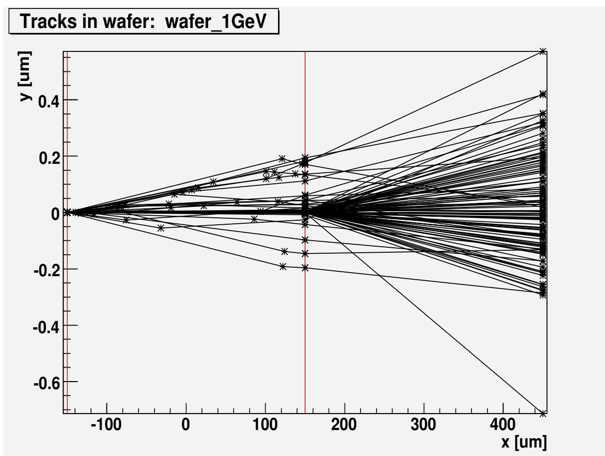


Figure 2: 100 tracks of 1 GeV electrons. The red lines represents front and the back surface of the wafer.

Output files were analyzed by the Root macro. A tangent of the scattering angle ϑ^{space} and its projection to x-y plane ϑ^{plane} is computed from the first step of the particle track after leaving the wafer. $\tan\vartheta^{space}$ and $\tan\vartheta^{plane}$ are given by the formulae:

$$\tan\vartheta^{space} = \sqrt{dy^2+dz^2}/dx \approx \vartheta^{space}, \tag{2}$$

$$\tan\vartheta^{plane} = dy/dx \approx \vartheta^{plane}, \tag{3}$$

where dx, dy and dz are differences of coordinates of end-points of the last step line.

A histogram was filled with the scattering angles ϑ to obtain an angular distribution.

2.1 Results of the model validation

The histogram of the projected angular distribution was fitted by a Gaussian distribution in a range given by $\pm 3/2$ of the histogram RMS. A width σ of the fitted Gaussian in a comparison with the theoretical value ϑ_0 is shown in Table 1. An example of the histogram is plotted in Fig. 3.

Data from Table 1 were displayed in Fig. 4.

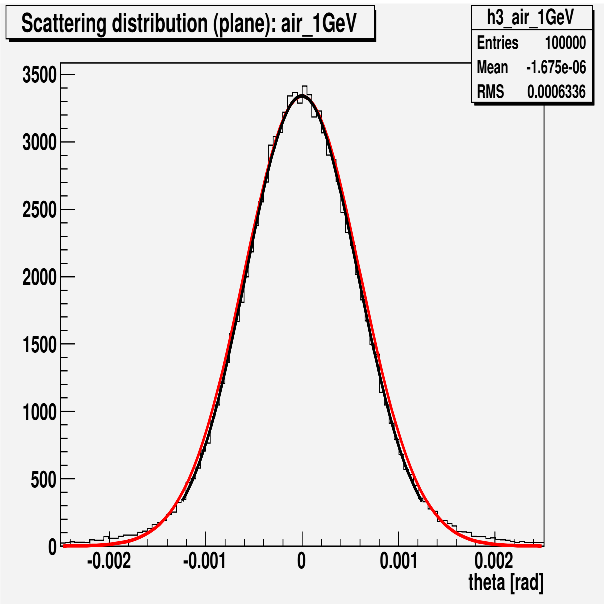


Figure 3: an example of the Gaussian fit of the projected angular distribution. The black line represents the Gaussian distribution fit, the red line represents the Gaussian part of the theoretical distribution.

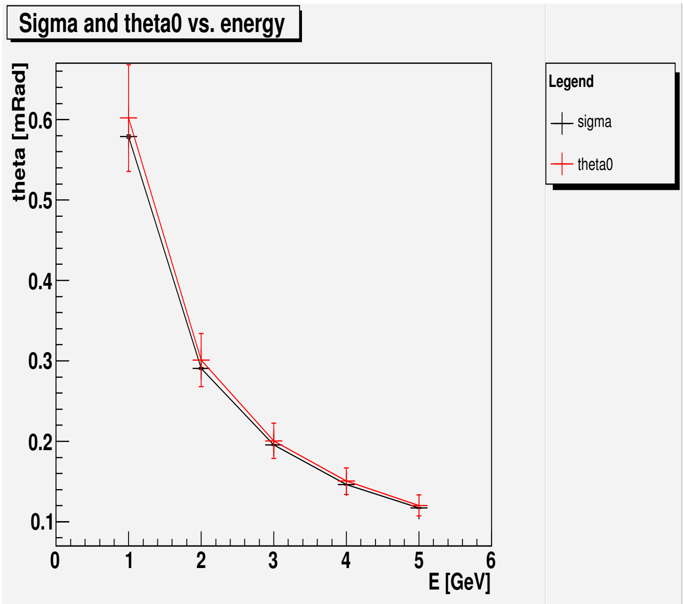


Figure 4: the value of ϑ_0 and σ vs. particle energy.

	Simulation	Theory	
E [GeV]	σ [mRad]	ϑ_0 [mRad]	ϑ_0/σ
1	0.57905 ± 0.00191	0.602 ± 0.066	0.96
2	0.29061 ± 0.00087	0.301 ± 0.033	0.97
3	0.19550 ± 0.00055	0.201 ± 0.022	0.97
4	0.14610 ± 0.00041	0.150 ± 0.017	0.97
5	0.11719 ± 0.00032	0.120 ± 0.013	0.97

Table 1: the comparison of σ with ϑ_0. An error of σ is a fit parameter error calculated by Root. According to [1], accuracy of ϑ_0 computed by formula (1) is 11% or better for $10^{-3} < x/X_0 < 100$. In our case $x/X_0 \approx 3 \cdot 10^{-3}$.

The width σ of the fitted Gaussian and the theoretical value ϑ_0 are the same in the range of their errors. From the histogram in Fig. 3 it's obvious that the Gaussian fit approximate well the simulated data.

3 Beam test simulation

Geometry

A geometry of the simulation is described in Fig. 5. The tested detector (DUT, Device Under Test) is placed in the center of the coordinate system. Two and two telescopes (TEL0 – TEL3) are placed in front of and at the back of the tested detector. The first and the last detector in the line is a scintilator (SCI0 and SCI1). Centers of the detectors are situated on the x-axis. a, b, c, d, e and f are distances between detectors centers. Results of the simulation for different values of these distances are presented below.

Telescopes and DUT wafers are made of silicon, windows are 50 μm copper foils. DUT silicon wafer is covered with a 0.2 μm aluminum layer. The scintilators are made of PMMA covered with a 75 μm aluminium layer.

An electron beam is parallel with the x-axis. It has a square profile with a 3 mm side and it is homogenous. Electrons move in a positive direction of the x-axis.

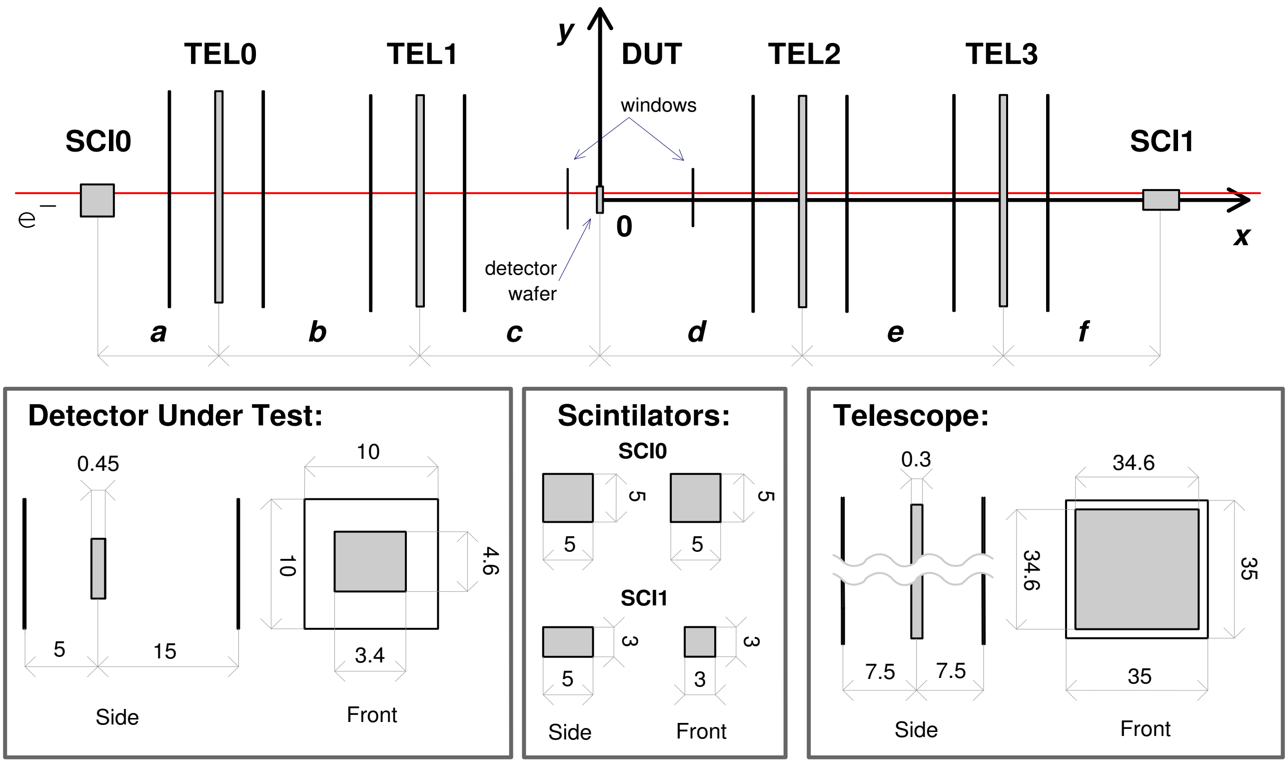


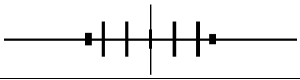
Figure 5: the geometry of the simulation. All dimensions in the figure are in milimeters.

Distances and window thicknesses used for the simulation are listes in Table 2.

Results of the beam test simulation

Results of the simulation of unscattered particles, en electron beam and a 180 GeV pion beam are presented in this paper. Four points of intersection in the telescopes that we obtained from the simulation were fitted by a straight line using the least square method. Distances between the actual and the fitted track (residuals) in DUT and the telescopes planes were put into histograms to get residual distributions.

In order to find a contribution of a telescopes intrinsic resolution to the residual distribution width we have fitted intersects of unscattered particles firstly. Unless the telescope intrinsic resolution is taken into account these points will lie on a straight line. Hence we have blurred the actual intersects with a Gaussian distribution in order to simulate the telescopes resolution. We

Geometry 1					
					
Distances					
a [mm]	b [mm]	c [mm]	d [mm]	e [mm]	f [mm]
15	40	40	40	40	15
Windows thickness					
1.	50 μm copper foils				
2.	no foils				
3.	150 μm copper foils				

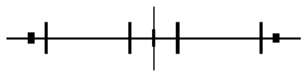
Geometry 2					
					
Distances					
a [mm]	b [mm]	c [mm]	d [mm]	e [mm]	f [mm]
15	140	40	40	140	15
Windows thickness					
1.	50 μm copper foils				

Table 2: distances and window thicknesses used for the simulation.

have set a width of the Gaussian to 2 μm. An example of such fit is shown in Fig. 6. To exclude bad fits χ^2 cuts were applied. In Table 3. there are widths of the residual distributions in the telescopes and the DUT planes for the two tested geometries. Just residuals in the x-y plane are shown here, the situation in the x-z plane is the same. In the DUT plane there is no significant difference between the tested geometries. χ^2 cuts also don't have any visible effect on the residual distribution width.

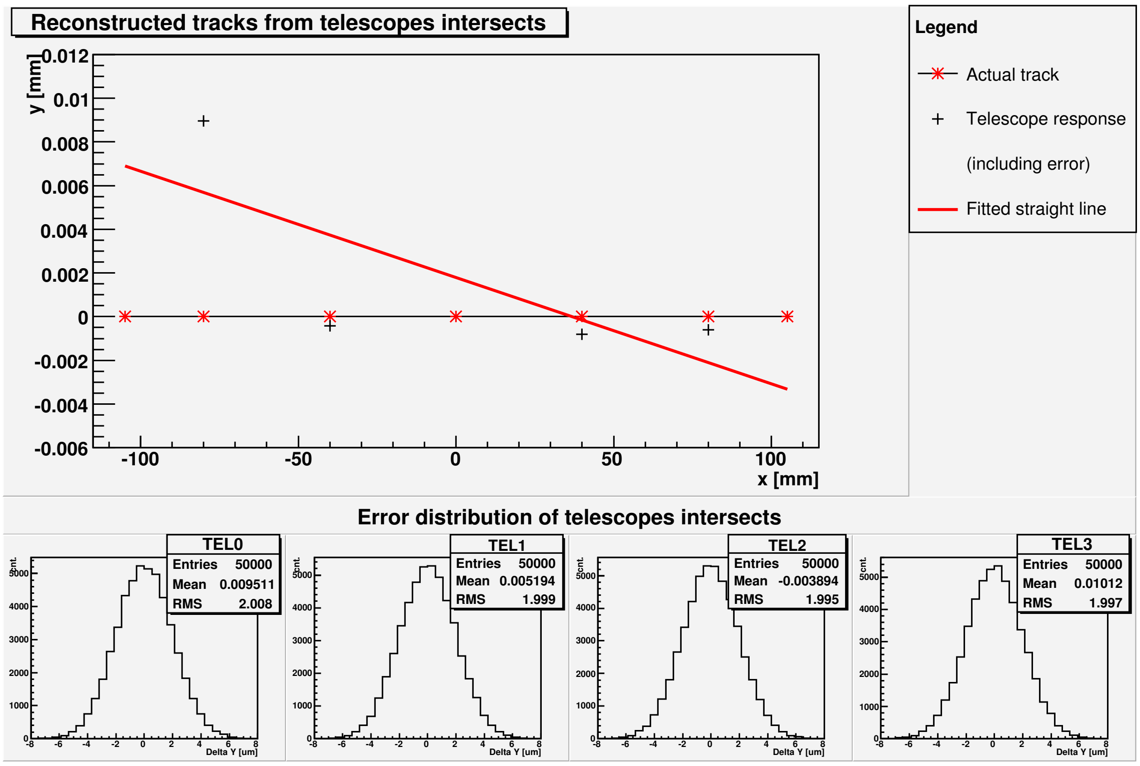


Figure 6: an example of an unscattered particle track fit in the x-y plane. The stars represents the actual intersects, the black crosses the telescopes response including the error. The residual is distance between the fitted line and the actual intersect in $x = 0$ plane.

After that, simulation with the electron beam were done. Beside telescopes intrinsic resolution there is also multiple scattering that significantly contribute to the residual distributions. Tracks of 1 GeV – 5 GeV electrons were simulated in G4, 50000 events for each run. A mean position in each detector wafer was stored in the file and analyzed. Particles that didn't hit the both scintillators were excluded from the analysis.

Beside the two geometries the three different window thicknesses were tested for the geometry 1 (see Tab. 2). An example of the residual distributions in the telescope and DUT planes is shown

Geometry 1, no. of events: 50000						
χ^2 cut		TEL0	TEL1	TEL2	TEL3	DUT
100%	$\sigma R(y)$ [μm]	1.183 ± 0.004	1.596 ± 0.005	1.592 ± 0.005	1.181 ± 0.004	0.991 ± 0.006
70%	$\sigma R(y)$ [μm]	0.822 ± 0.003	1.122 ± 0.004	1.118 ± 0.004	0.820 ± 0.003	0.993 ± 0.007
50%	$\sigma R(y)$ [μm]	0.654 ± 0.003	0.892 ± 0.004	0.888 ± 0.004	0.651 ± 0.003	0.992 ± 0.008
30%	$\sigma R(y)$ [μm]	0.483 ± 0.003	0.661 ± 0.004	0.653 ± 0.004	0.478 ± 0.003	0.99 ± 0.01
Geometry 2, no. of events: 50000						
χ^2 cut		TEL0	TEL1	TEL2	TEL3	DUT
100%	$\sigma R(y)$ [μm]	1.046 ± 0.003	1.677 ± 0.005	1.675 ± 0.005	1.045 ± 0.003	0.991 ± 0.006
70%	$\sigma R(y)$ [μm]	0.727 ± 0.003	1.187 ± 0.004	1.182 ± 0.004	0.726 ± 0.003	0.993 ± 0.007
50%	$\sigma R(y)$ [μm]	0.578 ± 0.003	0.941 ± 0.004	0.938 ± 0.004	0.577 ± 0.003	0.988 ± 0.008
30%	$\sigma R(y)$ [μm]	0.427 ± 0.002	0.700 ± 0.004	0.687 ± 0.004	0.422 ± 0.002	0.98 ± 0.01

Table 3: widths of the residual distributions of the unscattered particle for two geometries. χ^2 cuts were applied to exclude bad fits, hence the statistics were reduced to 70%, 50% and 30% of the original number of events.

Energy: 1 GeV, no. of events: 36486						
χ^2 cut [mm^2]		TEL0	TEL1	TEL2	TEL3	DUT
(100%)	$\sigma R(y)$ [μm]	22.36 ± 0.08	23.88 ± 0.09	24.96 ± 0.09	23.10 ± 0.09	38.8 ± 0.2
0.0025 (70%)	$\sigma R(y)$ [μm]	13.26 ± 0.06	16.08 ± 0.07	16.67 ± 0.07	13.63 ± 0.06	28.6 ± 0.2
0.0013 (50%)	$\sigma R(y)$ [μm]	9.89 ± 0.05	12.55 ± 0.07	12.83 ± 0.07	10.07 ± 0.05	22.9 ± 0.2
0.0006 (30%)	$\sigma R(y)$ [μm]	6.97 ± 0.05	9.17 ± 0.06	9.21 ± 0.06	7.00 ± 0.05	18.9 ± 0.2

Table 4: widths of the residual distributions of the 1 GeV electrons.

in Fig. 7, corresponding values are tabled in Table 4. Here the χ^2 cuts play an important role in reduction of an influence of the multiple scattering. A comparison of the two tested geometries is shown in the summary chart in Fig. 8 and in Table 5. Only the widths of the DUT residual distributions are shown in the figure. The thickness of the windows was 50 μm for the both geometries.

A comparison of the results for the different window thickness is shown in Fig 9. Since there is a lot of numbers we show just values with the most strict χ^2 cut in Table 6.

Finally the CERN 180 GeV pion beam were simulated. Only the two geometries were tested with pions. Since they have much lower multiple scattering than 5 GeV electrons do, there wouldn't be significant dependence on the windows thickness. Values of the residual distribution width in the DUT plane are tabled in Table 7. The difference between the values with the χ^2 cuts is washed out by the contribution of the telescopes intrinsic resolution since it is much higher then the multiple scattering. The values for Geometry 1 and 2 with the 30% χ^2 cut are the same in the range of their errors.

3.1 Conclusions

We have tested our Geant 4 model on a simple problem that was an electron scattering in the single silicon wafer. Obtained angular distribution is in a good agreement with the theoretical one (see Table 1).

For unscattered particles there is no significant difference between the geometry 1 and 2. The intrinsic resolution of the telescopes was 2 μm. The corresponding residual distribution width in the DUT plane was approximately $\sigma \approx 1\,\mu$m. In order to improve the final resolution we have applied the χ^2 cuts to exclude bad fits. For the unscattered particle it hasn't any significant effect on the residual distribution in the DUT plane (see Table 3). But for 1 GeV to 5 GeV electrons it can be reduced using this method.

Geometry 2 gives wider residual distributions due to a multiple scattering. For 5 GeV electrons and the 30% χ^2 cut $\sigma = 4.28\,\mu$m for the Geometry 1 and $\sigma = 5.94\,\mu$m for the Geometry 2. Values for the both geometries, all tested energies and χ^2 cuts are displayed in Table 5. Three different

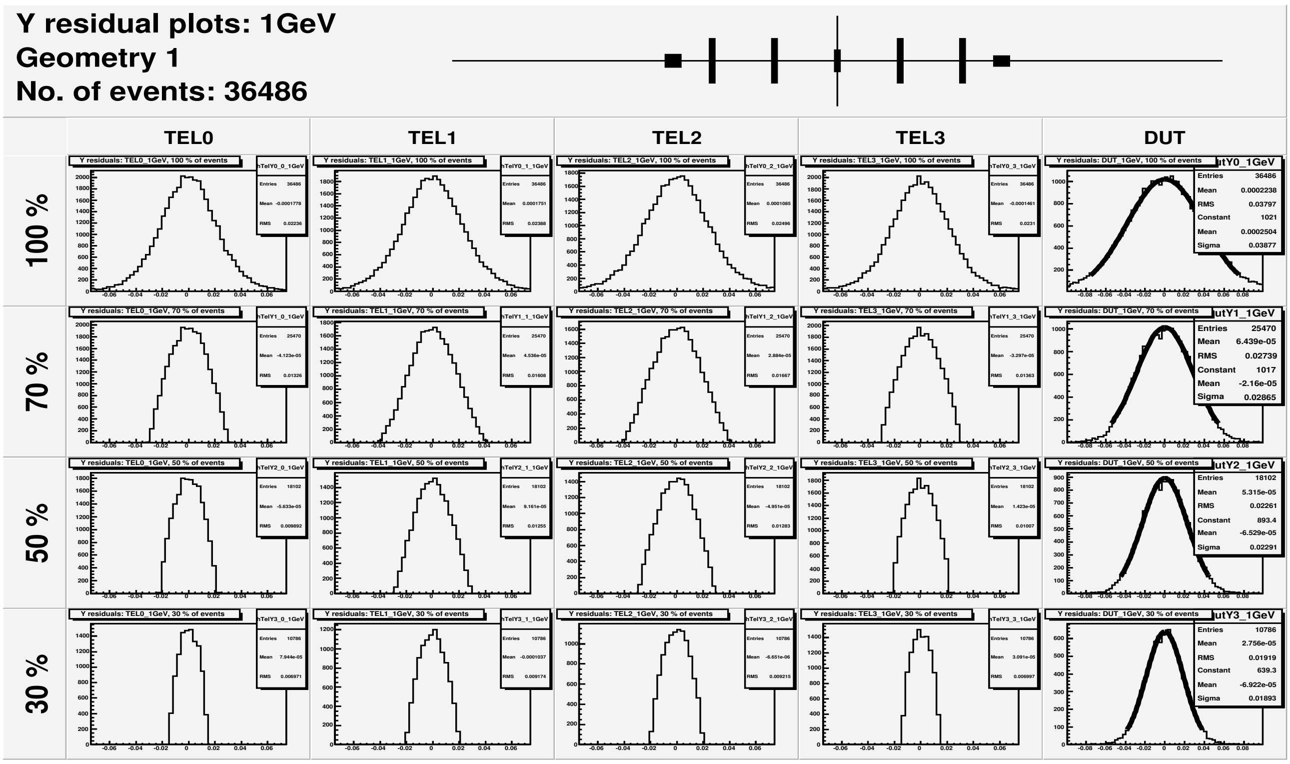


Figure 7: an example of the residual distributions of the 1 GeV elctrons. χ^2 cuts were applied to exclude bad fits. Note that the residual distributions in the telescopes planes are no more Gaussian after χ^2 cuts were applied. Hence we have used the histogram RMS instead of the Gaussian fit sigma as the distribution width parameter.

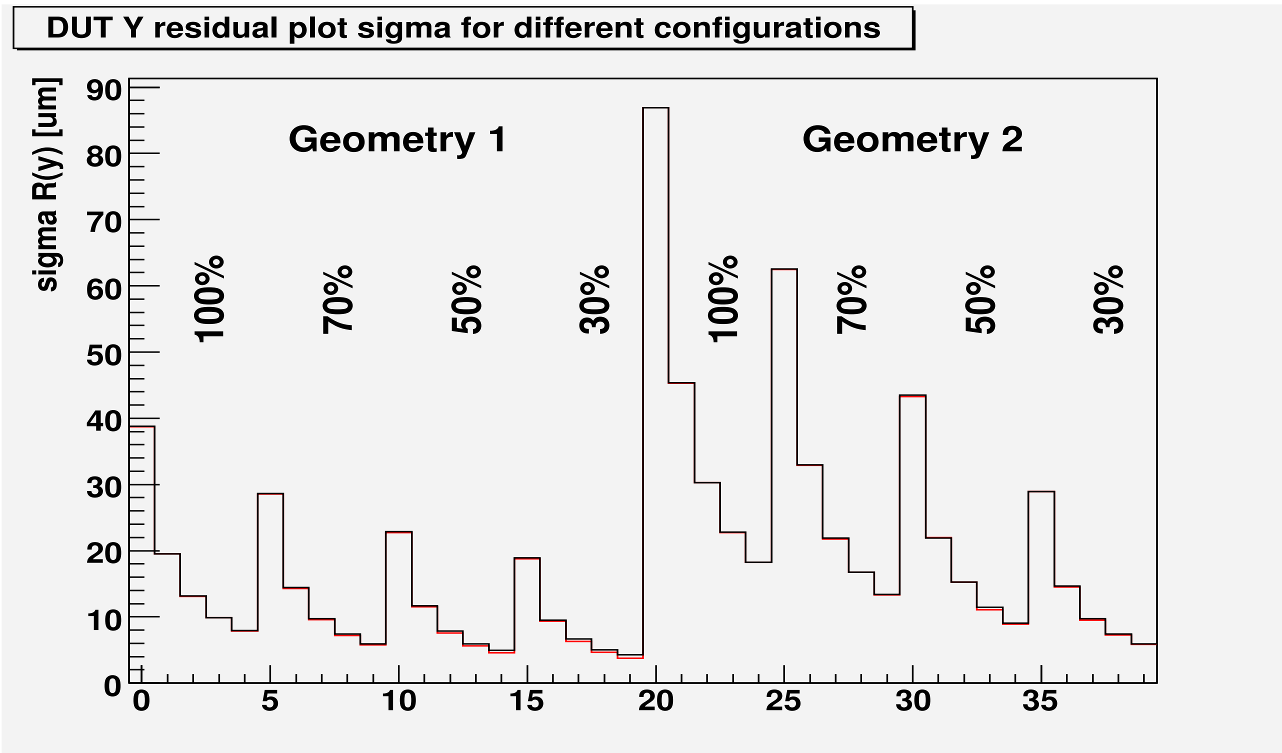


Figure 8: the comparison of the tested geometries for the electron beam. Each column represents value of the DUT residual distribution width. Groups of five columns labelled with 100% – 30% represent widths after the corresponding χ^2 cut. In each group the leftmost column corresponds to the 1 GeV electrons, the rightmost to the 5 GeV ones. The red line represents values for the ideal detectors, the black ones correspond to the simulation with the telescope intrinsic resolution included.

Geometry 1

Telescopes resolution included					
E [GeV]	1	2	3	4	5
100%	38.8 ± 0.2	19.6 ± 0.1	13.17 ± 0.07	9.90 ± 0.05	7.96 ± 0.04
70%	28.6 ± 0.2	14.41 ± 0.10	9.71 ± 0.06	7.38 ± 0.05	5.89 ± 0.04
50%	22.9 ± 0.2	11.69 ± 0.09	7.89 ± 0.06	5.90 ± 0.04	4.98 ± 0.04
30%	18.9 ± 0.2	9.48 ± 0.09	6.66 ± 0.06	5.02 ± 0.05	4.28 ± 0.04
Ideal telescopes					
E [GeV]	1	2	3	4	5
100%	38.7 ± 0.2	19.5 ± 0.1	13.11 ± 0.07	9.85 ± 0.05	7.90 ± 0.04
70%	28.5 ± 0.2	14.28 ± 0.10	9.56 ± 0.06	7.16 ± 0.05	5.76 ± 0.04
50%	22.8 ± 0.2	11.55 ± 0.09	7.57 ± 0.06	5.65 ± 0.04	4.60 ± 0.03
29%	18.8 ± 0.2	9.35 ± 0.09	6.30 ± 0.06	4.66 ± 0.05	3.75 ± 0.04

Geometry 2

Telescopes resolution included					
E [GeV]	1	2	3	4	5
100%	86.9 ± 0.6	45.4 ± 0.3	30.3 ± 0.2	22.8 ± 0.1	18.3 ± 0.1
70%	62.6 ± 0.6	33.0 ± 0.3	21.9 ± 0.2	16.8 ± 0.1	13.4 ± 0.1
50%	43.5 ± 0.5	21.9 ± 0.2	15.2 ± 0.1	11.47 ± 0.10	9.08 ± 0.08
30%	29.0 ± 0.4	14.7 ± 0.2	9.7 ± 0.1	7.44 ± 0.07	5.94 ± 0.06
Ideal telescopes					
E [GeV]	1	2	3	4	5
100%	86.9 ± 0.6	45.3 ± 0.3	30.3 ± 0.2	22.8 ± 0.1	18.2 ± 0.1
70%	62.5 ± 0.6	32.9 ± 0.3	21.7 ± 0.2	16.7 ± 0.1	13.3 ± 0.1
50%	43.3 ± 0.5	21.9 ± 0.2	15.3 ± 0.1	11.07 ± 0.09	8.91 ± 0.07
30%	28.9 ± 0.4	14.5 ± 0.2	9.54 ± 0.10	7.29 ± 0.07	5.85 ± 0.07

Table 5: the residual distribution width in the DUT plane for the electron beam. A comparison of the two geometries.

window thicknesses were tested too. For 5 GeV electrons and the 30% χ^2 cut there is approximately 1μm difference between the simulations with no module windows and the 50 μm copper windows. All values are tabled in Table 6.

The 180 GeV CERN pion beam was simulated at last. It has a significantly lower multiple scattering. Hence the main contribution to its residual distribution width come from the telescopes intrinsic resolution. It's approximately 1 μm for the both geometries.

References

[1] S. Eidelman *et al.*, Particle Physics Booklet, extracted from Review of Particle Physics, Physics Letters B592, 1, 2004

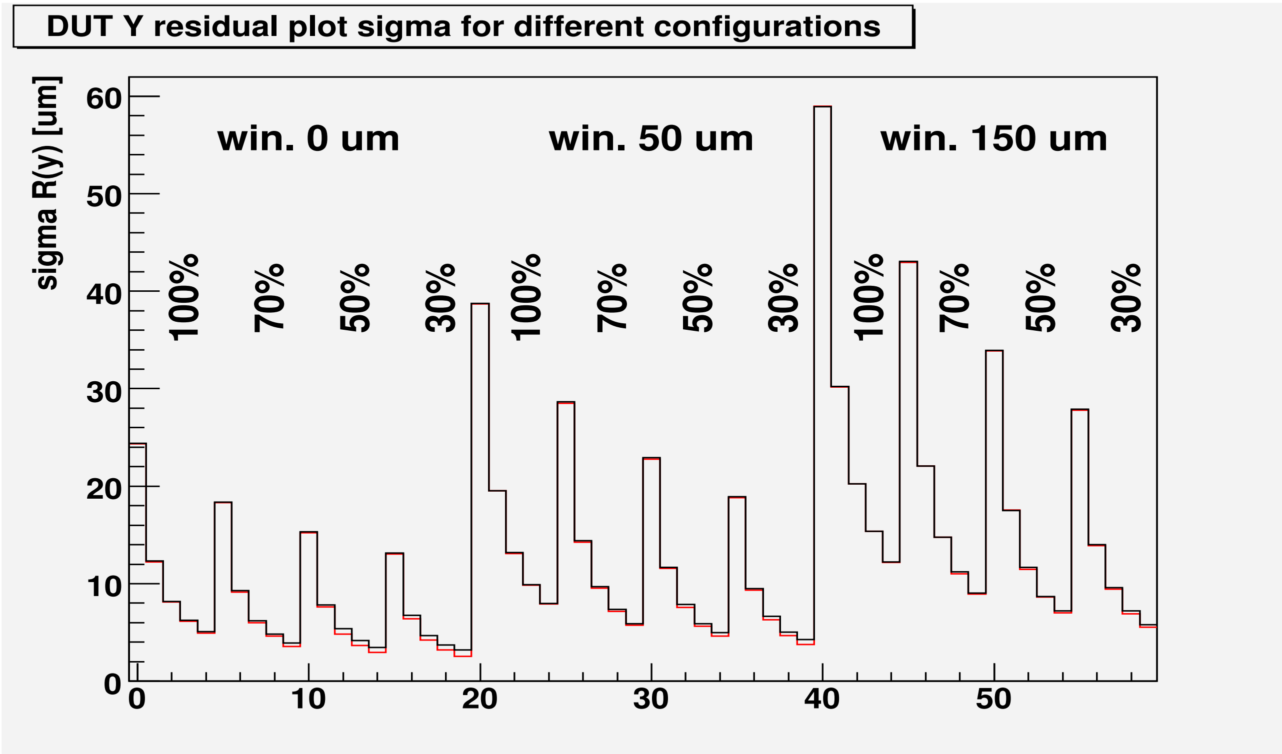


Figure 9: the comparison of the window thicknesses for the electron beam. The structure of the plot is the same as in Fig 8

Geometry 1

Telescopes resolution included					
E [GeV] / d [μm]	1	2	3	4	5
0	13.2 ± 0.1	6.77 ± 0.07	4.67 ± 0.05	3.73 ± 0.04	3.20 ± 0.03
50	18.9 ± 0.2	9.48 ± 0.09	6.66 ± 0.06	5.02 ± 0.05	4.28 ± 0.04
150	27.9 ± 0.3	14.0 ± 0.1	9.6 ± 0.1	7.21 ± 0.08	5.79 ± 0.06
Ideal telescopes					
0	13.0 ± 0.1	6.38 ± 0.06	4.23 ± 0.04	3.19 ± 0.03	2.55 ± 0.03
50	18.8 ± 0.2	9.35 ± 0.09	6.30 ± 0.06	4.66 ± 0.05	3.75 ± 0.04
150	27.8 ± 0.3	13.9 ± 0.1	9.44 ± 0.10	6.93 ± 0.07	5.55 ± 0.06

Table 6: the residual distribution width in the DUT plane for the electron beam. A comparison of the different window thicknesses d. Only values with the 30% χ^2 cut is shown in this table.

Geometry 1

Telescopes resolution included		
χ^2 cut		DUT
100%	$\sigma R(y)$ [μm]	1.020 ± 0.006
70%	$\sigma R(y)$ [μm]	1.025 ± 0.007
50%	$\sigma R(y)$ [μm]	1.024 ± 0.009
30%	$\sigma R(y)$ [μm]	1.02 ± 0.01
Ideal telescopes		
100%	$\sigma R(y)$ [μm]	0.215 ± 0.001
70%	$\sigma R(y)$ [μm]	0.1482 ± 0.0009
50%	$\sigma R(y)$ [μm]	0.1137 ± 0.0009
30%	$\sigma R(y)$ [μm]	0.0956 ± 0.0010

Geometry 2

Telescopes resolution included		
χ^2 cut		DUT
100%	$\sigma R(y)$ [μm]	1.138 ± 0.007
70%	$\sigma R(y)$ [μm]	1.127 ± 0.008
50%	$\sigma R(y)$ [μm]	1.132 ± 0.009
30%	$\sigma R(y)$ [μm]	1.12 ± 0.01
Ideal telescopes		
100%	$\sigma R(y)$ [μm]	0.499 ± 0.003
70%	$\sigma R(y)$ [μm]	0.355 ± 0.003
50%	$\sigma R(y)$ [μm]	0.238 ± 0.002
30%	$\sigma R(y)$ [μm]	0.133 ± 0.001

Table 7: the widths of the residual distributions in DUT for the 180 GeV pion beam.